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Cosmological Relativity: Determining the Universe by the Cosmological Redshift as Infinite and Curved

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Using cosmological relativity theory, we derive the formula for the cosmological redshift written explicitly in terms of $1 - \Omega$, where $\Omega = \rho/\rho_c$ is the ratio of the average mass density to the critical "closure" density. Based on the present day data of observed redshifts, we conclude that $\Omega < 1$, which means the universe is infinite and curved, and expands forever.

1. INTRODUCTION

In spite of the advances made in recent years in cosmology, the question of what kind of universe we live in is still unsettled (Dekel, 1994; Gunn, 1986; Loh, 1986; Loh and Spillar, 1986a,b; Peccei, 1993; Peebles, 1993; Rees, 1993; Riordan and Schramm, 1993; Turner, 1986). According to FRW, based on general relativity theory, one can calculate a critical mass density ρ_c at the present time such that the Hubble expansion will ultimately be reversed if and only if the actual average mass density ρ exceeds ρ_c, where $\rho_c = 3H_0^2/8\pi G$, with H_0 the present-time Hubble parameter and G the Newtonian gravitational constant. The value of ρ_c is about 10^{-29}g/cm^3, a few hydrogen atoms per cubic meter throughout the cosmos. Thus, it is the value of $\Omega\,(=\rho/\rho_c)$ that determines the behavior of the expansion of the universe: $\Omega > 1$, a finite universe; $\Omega < 1$, an infinite curved universe; $\Omega = 1$, an infinite flat-space universe; and the sign of the quantity $1 - \Omega$ is the determining factor here (Ohanian and Ruffini, 1994; Landau and Lifshitz, 1979).

In this paper we use cosmological general relativity theory (Behar and Carmeli, 2000; Carmeli, 1996, 1997) to derive a general formula for the redshift in which the term $1 - \Omega$ appears explicitly. Since there are enough data of measurements of redshifts, this allows one to determine what the sign of $1 - \Omega$ is, positive, zero, or negative. Our conclusion is that $1 - \Omega$ cannot be negative or

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zero. This means that the universe is infinite and curved and expands forever, a result favored by some cosmologists (Turner, 1993). To this end we proceed as follows.

2. GRAVITATIONAL FIELD EQUATIONS

We seek a spherical symmetric solution to the Einstein field equations $G_\mu^\nu = \kappa T_\mu^\nu$ and use spherical coordinates $x^\mu = (x^0, x^1, x^2, x^3) = (\tau v, r, \theta, \phi)$, where τ is Hubble's time in the zero-gravity limit and v is the velocity parameter. Since the universe is spherically symmetric at any chosen point, the line element we seek is of the form

$$ds^2 = \tau^2 dv^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\phi^2), \tag{1}$$

where comoving coordinates, as in the Friedmann theory, are used, and λ is a function of the radial distance r only. To determine λ it is enough to solve the field equation $G_0^0 = \kappa T_0^0$ which turns out to be (Behar and Carmeli, 2000; Carmeli, 1996, 1997)

$$G_0^0 = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = \frac{8\pi G}{c^4} T_0^0 = \frac{8\pi G}{c^2}\rho_{\text{eff}}, \tag{2}$$

where a prime denotes derivation with respect to r and $\rho_{\text{eff}} = \rho - \rho_{\text{c}}$. The solution of Eq. (2) is

$$e^{-\lambda} = 1 + \frac{(1-\Omega)}{c^2\tau^2} r^2 = 1 + \frac{r^2}{a^2}, \tag{3}$$

with $g_{11} = -e^\lambda$, $a^2 = c^2\tau^2/(1-\Omega)$, and $\Omega = \rho/\rho_{\text{c}}$.

3. COSMOLOGICAL REDSHIFT

Having the metric tensor we may now find the redshift of light emitted in the cosmos. As usual, at two points 1 and 2 we have for the wavelengths,

$$\frac{\lambda_2}{\lambda_1} = \frac{ds(1)}{ds(2)} = \sqrt{\frac{g_{11}(1)}{g_{11}(2)}}. \tag{4}$$

Using now the solution for g_{11} in Eq. (4) we obtain

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{1 + \frac{r_2^2}{a^2}}{1 + \frac{r_1^2}{a^2}}}. \tag{5}$$

For a sunlike body located at the coordinates origin, and an observer at a distance r from the center of the body, we have $r_2 = r$ and $r_1 = 0$, thus

$$\frac{\lambda_2}{\lambda_1} = \sqrt{1 + \frac{r^2}{a^2}} = \sqrt{1 + \frac{(1-\Omega)r^2}{c^2\tau^2}} \tag{6}$$

for the cosmological contribution to the redshift. If, furthermore, $r \ll a$ we have

$$\frac{\lambda_2}{\lambda_1} = 1 + \frac{r^2}{2a^2} = 1 + \frac{(1-\Omega)r^2}{2c^2\tau^2}, \tag{7}$$

to the lowest apprixomation in r^2/a^2, and thus

$$z = \frac{\lambda_2}{\lambda_1} - 1 = \frac{r^2}{2a^2} = \frac{(1-\Omega)r^2}{2c^2\tau^2}. \tag{8}$$

From Eqs. (6)–(8) it is clear that Ω cannot be larger than 1 since otherwise z will be negative, which means blueshift, and as is well-known, nobody sees such a thing. If $\Omega = 1$, $z = 0$, and for $\Omega < 1$ we have $z > 0$. The case of $\Omega = 1$ is also implausible since the light from stars we see is usually redshifted more than the redshift due to the gravity of the body emitting the radiation, as is evident from our sun, for example, whose emitted light is shifted only by $z = 2.12 \times 10^{-16}$ (Carmeli, 1982).

4. CONCLUSIONS

One can thus conclude that the theory of cosmological general relativity implies that the universe is infinite and curved and expands forever. As is well-known the standard FRW model does not relate the cosmological redshift to the kind of the universe. Our conclusion is also in full agreement with the measurements recently obtained by the *High-Z Supernovae Team* and the *Suprenovae Cosmology Project* (Garnavich *et al.*, 1998a,b; Perlmutter *et al.*, 1997, 1998, 1999; Riess *et al.*, 1998; Schmidt *et al.*, 1998).

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